

Topics in General Equilibrium Theory

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Review

- Smith
- Nassau Senior
- J. S. Mill
- Hayek
- Robbins

Walrasian General Equilibrium

1 Introduction

- What is equilibrium in chemistry?

Misconception 1: Many students erroneously assume that the condition of equilibrium means equal concentrations of reactants and products.

Misconception 2: Many students believe that a reaction at equilibrium has stopped.

Misconception 3: Students do not easily grasp the notion that one can approach an equilibrium state from either direction.

- Why is it hard to teach in economics?

- ## 2 participation exercise on excess demand and interdependent markets: tatonnement.

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Equilibrium in chemistry?

Gibbs [1928]: p 's and q 's

6

VARIATION OF THE

or more briefly by

$$D dp_1 \dots dp_n dq_1 \dots dq_n, \quad (11)$$

where D is a function of the p 's and q 's and in general of t also, for as time goes on, and the individual systems change their phases, the distribution of the ensemble in phase will in general vary. In special cases, the distribution in phase will remain unchanged. These are cases of *statistical equilibrium*.

Equilibrium in chemistry?

Gibbs [1928]: p's and q's

DENSITY-IN-PHASE.

7

In the first place, let us consider the number of systems which in the time dt pass into or out of the specified element by p_1 passing the limit p_1' . It will be convenient, and it is evidently allowable, to suppose dt so small that the quantities $\dot{p}_1 dt, \dot{q}_1 dt$, etc., which represent the increments of p_1, q_1 , etc., in the time dt shall be infinitely small in comparison with the infinitesimal differences $p_1'' - p_1', q_1'' - q_1'$, etc., which determine the magnitude of the element of extension-in-phase.

Equilibrium in chemistry?

Gibbs [1928]: p 's and q 's

entiation. The condition of statistical equilibrium is therefore

$$\sum \left(\frac{dD}{dp_1} \dot{p}_1 + \frac{dD}{dq_1} \dot{q}_1 \right) = 0. \quad (20)$$

If at any instant this condition is fulfilled for all values of the p 's and q 's, $(dD/dt)_{p,q}$ vanishes, and therefore the condition will continue to hold, and the distribution in phase will be permanent, so long as the external coördinates remain constant.

Why is it hard to teach in economics?

Samuelson [1947]: x's and y's

- 1 Samuelson aspired to raise the prestige of economics by making it more like physics.
- 2 Samuelson attempted to purge ideas such as “cause” and “meaning” from economic thinking.
- 3 Samuelson purposely obscured his borrowing from Gibbs. See also McCloskey [1985].

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Why is it hard to teach in economics? Samuelson [1947]: x's and y's

REVIEW BY GERHARD TINTNER

*Research Associate, Department of Applied Economics
Cambridge University, Cambridge, England*

*ON LEAVE: Professor of Economics and Mathematics
Iowa State College, Ames, Iowa*

THIS is a very important contribution in the field of pure economics which ought to be of interest to econometricians and those statisticians who concern themselves with economic matters.

The author endeavors throughout to derive “operationally meaningful” propositions in economics from the assumption of maximizing behavior. Operationally meaningful statements are hypotheses about empirical data which could conceivably be refuted, if only under ideal conditions. These ideas are related to those of some modern positivist philosophers, especially Bridgeman. It is perhaps unfortunate that the author has not included a methodological chapter in his book. An explicit statement of his philosophical ideas may have contributed to the understanding of his book. It may have helped some economists who are sceptical about the applicability of radical positivism and behaviorism in the social sciences.

Why is it hard to teach in economics?

Samuelson [1947]: x's and y's

My copy of Bridgman is probably the one Tintner read at Iowa State.

Tatonnement

Class participation exercise:

- 1 Name a product you have recently bought or might buy this week.
- 2 Class name the lowest price you have seen for that product.
- 3 Helper write $p_n = \text{price}$ for each of $1..n$ products.
- 4 At the prices named, would the student naming that product buy it or not?
- 5 In the market for each product we will have positive or negative excess demand.
- 6 How will changing demand for one product affect how much we can buy of another?
- 7 We can imagine in which direction to adjust the prices to reduce excess demand, giving a sense of the tatonnement process envisioned by Walras.

Tatonnement

- 1 Will the announced price vector clear the markets?
- 2 Walras thought so— assuming linear equations.
- 3 $x=3+2$
- 4 $y=x+1$
- 5 Not as easy as counting equations.

Utility and production functions

$$u(x_1, x_2, \dots, x_N) = \left[\sum_{i=1}^N \alpha_i x_i^\rho \right]^{\frac{1}{\rho}}$$

Where

N = Number of Private Goods

x_j = Quantity of Private Good g Consumed

α 's = Constant Elasticity of Scale consumption coefficients

ρ =

$$\frac{\sigma - 1}{\sigma}$$

σ = Elasticity of Substitution

Simulation

An economy with supply and demand of goods and the inputs needed to make them.

Competitive Equilibrium - Version 2012.11.09: Simulation-1.fxc

File Options Help

Number of Private ... 4 Number of House... 2 Number of Private ... 2

Public G... Externality Good Names Production Fu...

Stop After Each Major ... Utility Functi... Taxes Ad Val

Endowments Taxes Unit

Return To Spreadsheet

Demand: Household

Good		1	2
1	X	15.70	8.06
2	Y	13.51	11.57
3	L	9.19	5.90
4	K	.00	.00
Inc		11.40	7.32
Tran		.00	.00
Util		13.48	8.75

Summary

Good		Price	Demand	Supply	ExDem
1	X	.3629	23.76	23.76	.00
2	Y	.2530	25.08	25.08	.00
3	L	.2481	48.00	48.00	.00
4	K	.1360	50.00	50.00	.00

Select All: Ctrl-A Copy: Ctrl-C Numeraire

(Not a monetary economy)

Note, in this simulation, taxes don't destroy money. Prices are relative and thus should be thought of in terms of 'smoked haddock' (a numeraire good) rather than dollars.

Simulation

Elast is σ , other values are α 's. Same in the production function panel, which we ignore in this simple case. Let's add a household that desires only one of the goods.

Utility	House 1	House 2	House 3
Good 1: X	.50	.40	
Good 2: Y	.30	.40	1.00
Good 3: L	.20	.20	.20
Good 4: K			
Elast	1.00	1.00	1.00

Simulation

Give the added household exactly the same endowment as the first household.

Competitive Equilibrium - Version 2012.11.09: Simulation-2.fxc

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Find Equilibrium

Endow	House 1	House 2	House 3
Good 1: X			
Good 2: Y			
Good 3: L	24.00	24.00	24.00
Good 4: K	40.00	10.00	40.00
Redist	.33	.33	.33

Simulation

Unexpected result? Partial equilibrium thinking does not apply when markets are interconnected.

Competitive Equilibrium - Version 2012.11.09: Simulation-2.fxc

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Return To Spreadsheet

Demand: Household

Good		1	2	3
1	X	12.10	5.84	.00
2	Y	13.33	10.73	37.03
3	L	12.88	7.77	10.73
4	K	.00	.00	.00
Inc		10.80	6.52	10.80
Tran		1.07	1.07	1.07
Util		12.61	7.89	59.52

Summary

Good		Price	Demand	Supply	ExDem
1	X	.4465	17.94	17.94	.00
2	Y	.2431	61.09	61.09	.00
3	L	.1678	72.00	72.00	.00
4	K	.1427	90.00	90.00	.00

Select All: Ctrl-A Copy: Ctrl-C Numeraire

Simulation

Unexpected result? Partial equilibrium thinking does not apply when markets are interconnected.

Competitive Equilibrium - Version 2012.11.09: Simulation-1.fxc

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Applied General Equilibrium

The preceding was not Tatonnement.

To find a fixed point, $p_1^*..p_n^*$, where the inspection of excess demand would lead you to call out the same set of p 's, is a difficulty problem that the Tatonnement procedure, or algorithm, can't solve. For a solution, imagine using triangle to represent two prices.

Applied General Equilibrium

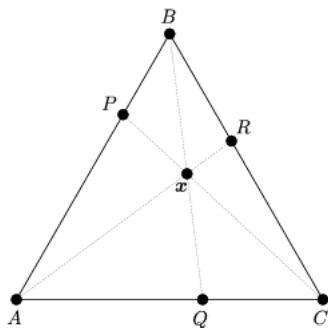


Figure: Ternary plot of $x = (p_1, p_2)$

Applied General Equilibrium

Herbert Scarf, July 25, 1930 to November 15, 2015.

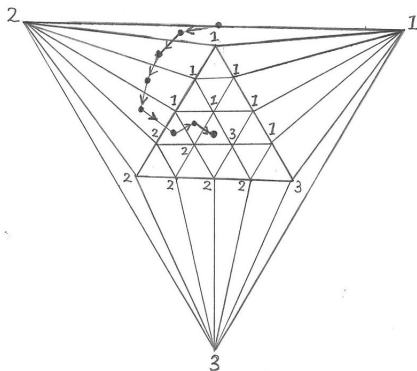


Figure: Scarf Algorithm: add a fine grid of smaller triangles, imagine rooms you exit through a door you didn't enter until you find the fixed point.

Complete Markets

- So far, we have probably been thinking only in terms of rearranging who owns goods that exist today. We can, however, imagine markets across space and time, products like contracts for apples tomorrow in Chicago, instead of just apples.

Complete Markets

- One of the clearest introductions I have seen is a 1990's paper from the St. Louis Fed, Flood et al. [1991].
- The Spirit of the 90's

“Another woman at the firm, Ros Stephenson—perhaps the only Lehman banker besides Fuld who could get Kohlberg Kravis Roberts kingpin Henry Kravis on the phone—was furious about the appointment and took her complaint directly to Dick Fuld, who, as always, backed Gregory.”
—*Too Big to Fail*

- This quote from the *Too Big to Fail* account of derivatives today hearkens back to the simpler *Barbarians at the Gate* days of leveraged buyouts (Kohlberg Kravis Roberts) during the dawn of financial engineering.

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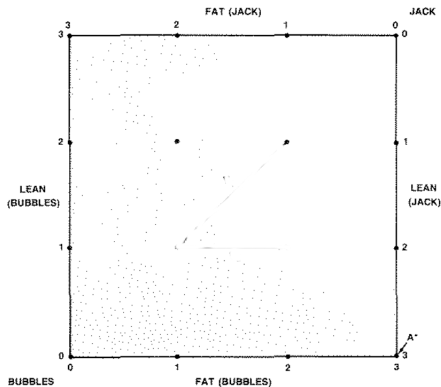
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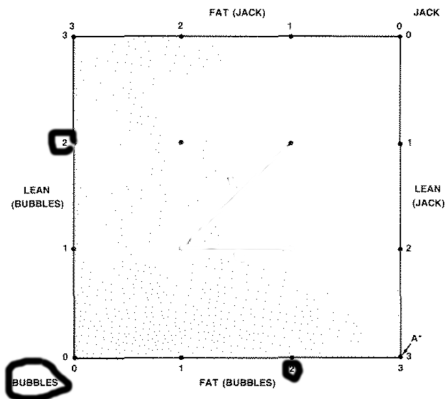
Complete Markets Basic Example

Figure 1
The Allocation of Pork



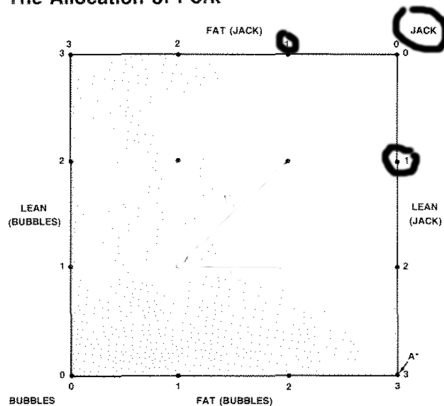
Complete Margets Basic Example

Figure 1
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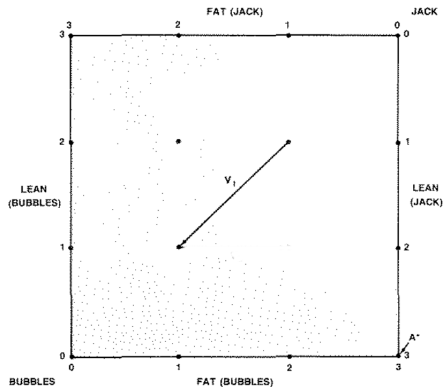


Complete Markets Basic Example

	COMMODITY	
PRICE	FAT	LEAN
\$P	1	1

Complete Margets Basic Example

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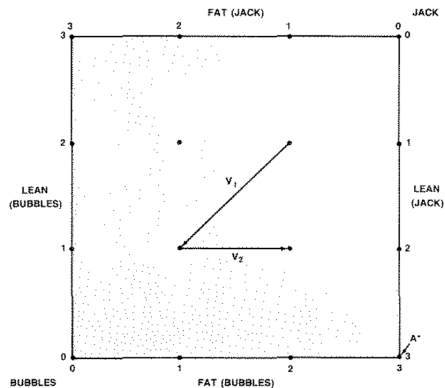


Complete Markets Basic Example

PRICE	COMMODITY	
	FAT	LEAN
\$P	1	1
$\$(P/2)$	1	0

Complete Markets Basic Example

Figure 1
The Allocation of Pork



Complete Markets are Good

“The common implication in each example is that additional markets can improve the welfare of all concerned. That is, given the ability to reallocate, individuals will do so: they will exchange relatively less desirable commodity bundles for those that are, for them, relatively more desirable. A complete system of markets provides this ability.”

So let's engineer derivatives and gamble

“While speculation in financial markets may or may not be unfettered gambling, the implicit transfer of risk from hedgers to speculators still produces economic value. The theory of complete markets thus provides a systematic explanation for the popularity and value of many so-called derivative securities, such as futures and options.”

"Not so fast," says Aunt Deirdre.

"Like the old Marxists, and the older Christians, the neo-institutionalists among Samuelsonian economists want a theory that would, if it were true, have allowed them in 1700 to lay down the future."

"Meaning matters, metaphors matter, stories matter, identity matters, ethics matter. Considering that we are humans, not grass, it matters a great deal."

Max U vs. Humanomics: A Critique of Neo-Institutionalism. Dierdre McCloskey, forthcoming, Journal of Institutional Economics.

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